Chapter 2

Queueing Theory and Simulation

Based on the slides of Dr. Dharma P. Agrawal, University of Cincinnati and Dr. Hiroyuki Ohsaki Graduate School of Information Science & Technology, Osaka University, Japan

Introduction

- Several factors influence the performance of wireless systems:
  - Density of mobile users
  - Cell size
  - Moving direction and speed of users (Mobility models)
  - Call rate, call duration
  - Interference, etc.
- Probability, statistics theory ,traffic patterns, queueing theory, and simulation help make these factors tractable

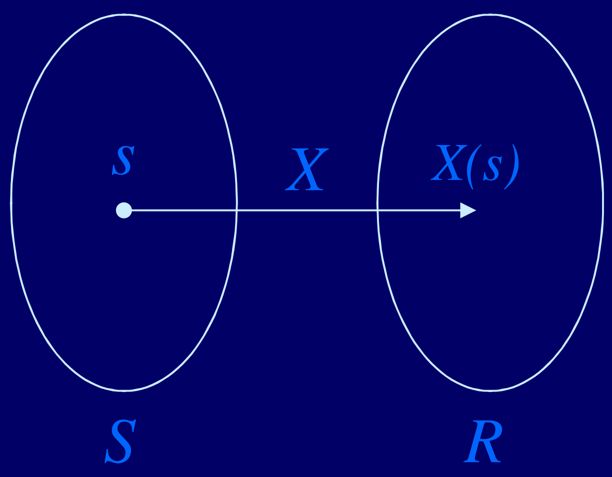
Outline

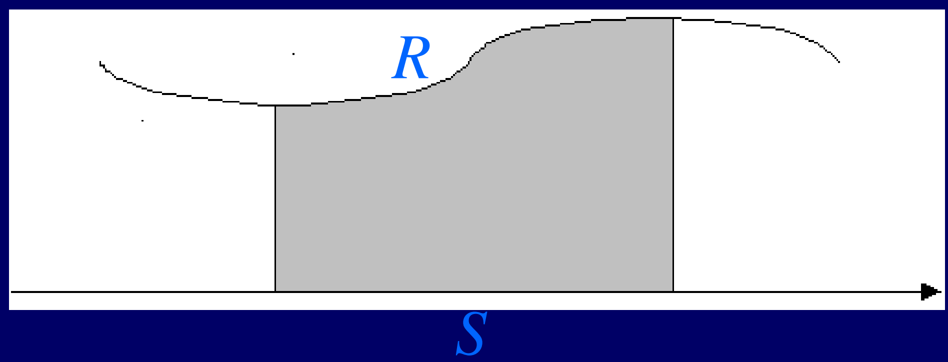
- Introduction
- Probability Theory and Statistics Theory
  - Random variables
  - Probability mass function (pmf)
  - Probability density function (pdf)
  - Cumulative distribution function (cdf)
  - Expected value, n^{th} moment, n^{th} central moment, and variance
  - Some important distributions
- Traffic Theory
  - Poisson arrival model, etc.
- Basic Queuing Systems
  - Little′s law
  - Basic queuing models
- Simulation

Background: Probability & Statistics

Probability Theory and Statistics Theory

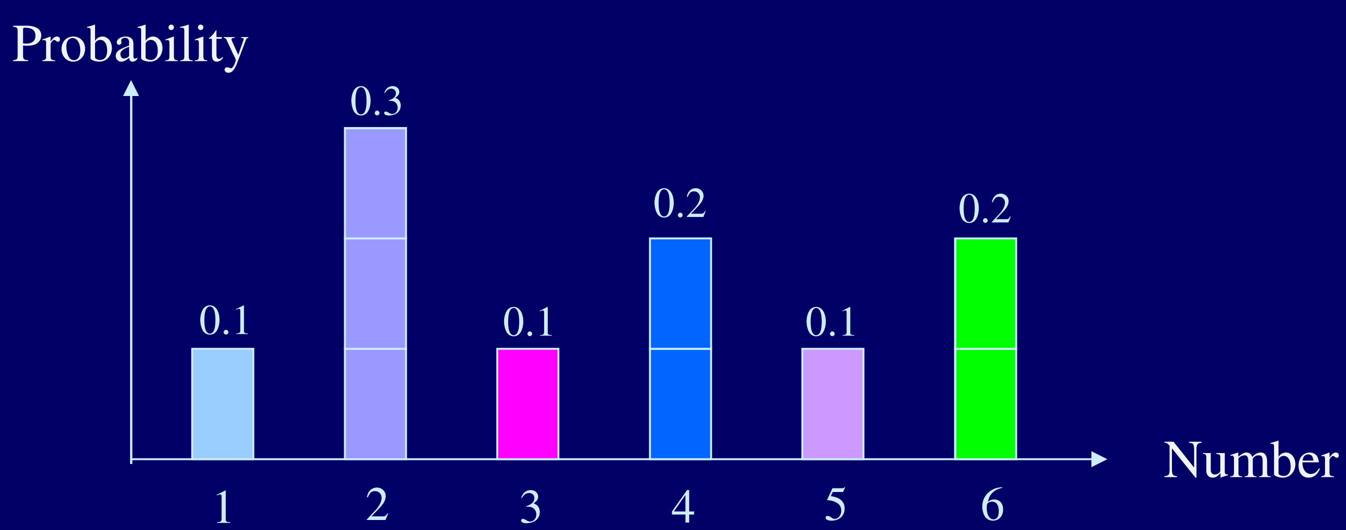
- A Random Variable (RV) provides a numerical description of a trial
- Random Variables (RVs)
  - Let *S* be the sample associated with experiment *E*
  - *X* is a function that associates a real number to each $s \in S$
  - RVs can be of two types: Discrete or Continuous
  - Discrete random variable => probability mass function (pmf)
  - Continuous random variable => probability density function (pdf)





Discrete Random Variables

- In this case, *X(s)* contains a finite or infinite number of values
  - The possible values of *X* can be enumerated
- E.g., throw a 6 sided dice and calculate the probability of a particular number appearing.



Discrete Random Variables

- The probability mass function (pmf) $p(k)$ of X is defined as:

  $$p(k) = p(X = k), \quad for\ k = 0, 1, 2, \ldots$$

  where

  1. Probability of each state occurring

     $0 \leq p(k) \leq 1$, for every k;

  2. Sum of all states

     $\sum p(k) = 1$, for all k.

Continuous Random Variables

- In this case, X contains an infinite number of values.
- E.g., spinning a pointer around a circle and measuring the angle it makes when it stops.
- E.g., height of a person in feet.
- Mathematically, X is a continuous random variable if there is a function f, called probability density function (pdf) of X that satisfies the following criteria:

  1. $f(x) \geq 0$, for all x;

  2. $\int f(x)dx = 1$.

Cumulative Distribution Function

- Applies to all random variables
- A cumulative distribution function (cdf) is defined as:
  - For discrete random variables:

$$P(k) = P(X \leq k) = \sum_{\text{all} \leq k} P(X = k)$$

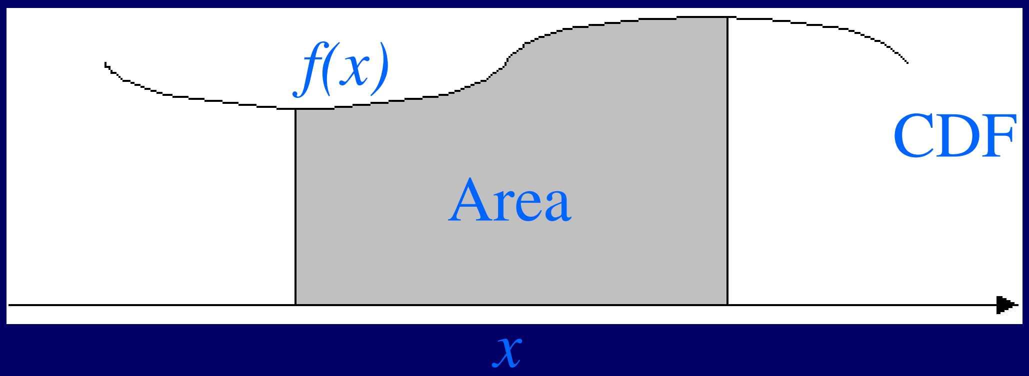
  - For continuous random variables:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx$$

Probability Density Function

- The pdf *f(x)* of a continuous random variable *X* is the derivative of the cdf *F(x)*, i.e.,

$$f(x) = \frac{dF_X(x)}{dx}$$



Expected Value, nth Moment, nth Central Moment, and Variance

- Discrete Random Variables
  - Expected value represented by E or average of random variable

$$E[X] = \sum_{\text{all} \le k} kP(X = k)$$

  - nth moment

$$E[X^n] = \sum_{\text{all} \le k} k^n P(X = k)$$

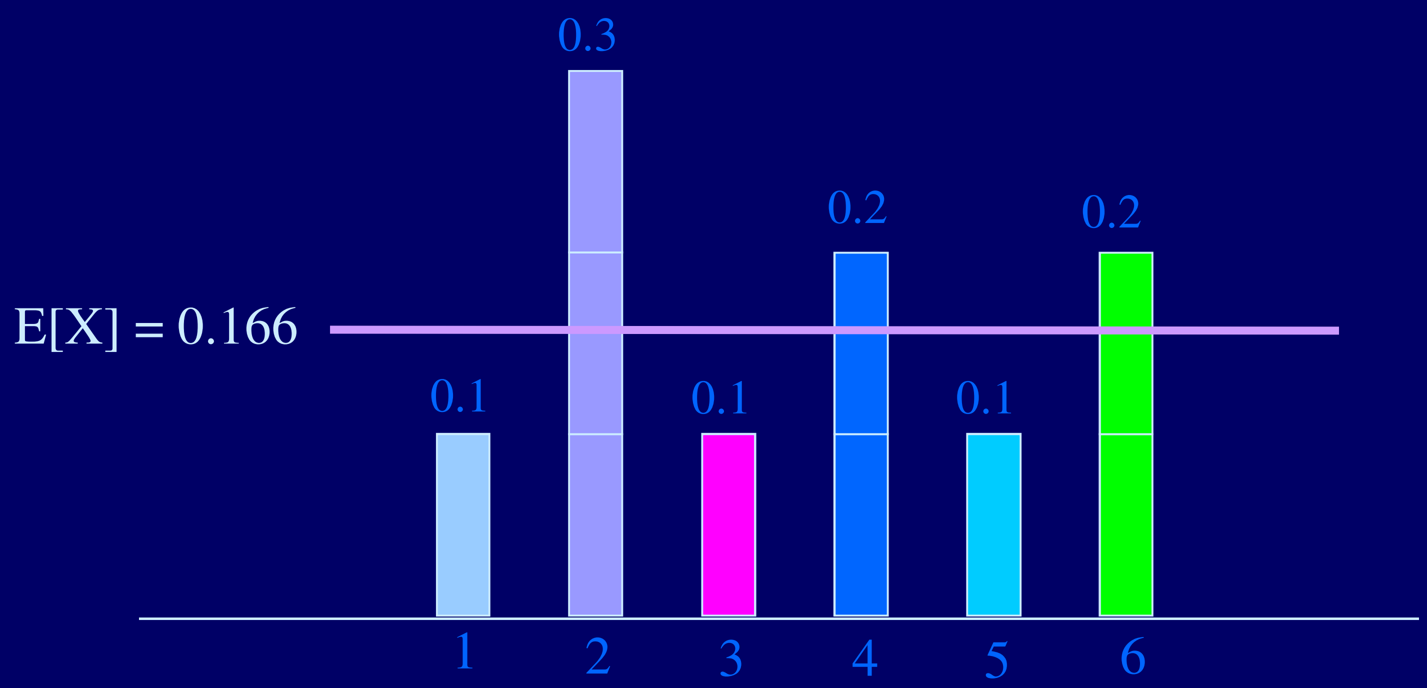
  - nth central moment

$$E[(X - E[X])^n] = \sum_{\text{all} \le k} (k - E[X])^n P(X = k)$$

  - Variance or the second central moment

$$\sigma^2 = Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

Expected Value, n^{th} Moment, n^{th} Central Moment, and Variance



Expected Value, nth Moment, nth Central Moment, and Variance

- Continuous Random Variable
  - Expected value or mean value

$$E[X] = \int_{-\infty}^{+\infty} x f(x) dx$$

  - nth moment

$$E[X^n] = \int_{-\infty}^{+\infty} x^n f(x) dx$$

  - nth central moment

$$E[(X - E[X])^n] = \int_{-\infty}^{+\infty} (x - E[X])^n f(x) dx$$

  - Variance or the second central moment

$$\sigma^2 = Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

Some Important Discrete Random Distributions

- Bernoulli
  - A classical example of a Bernoulli experiment is a single toss of a coin. The coin might come up heads with probability p and tails with probability q=1-p. The experiment is called fair if if both possible outcomes have the same probability.
  - The probability mass function of this distribution is

$$f(k;p) = \begin{cases} p & \text{if } k = 1, \\ 1-p & \text{if } k = 0. \end{cases}$$

Some Important Discrete Random Distributions

- Poisson

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

  - $E[X] = \lambda$, and $Var(X) = \lambda$

- Geometric

  $P(X = k) = p(1\text{-}p)^{k\text{-}1}$,

  where p is success probability

  - $E[X] = 1/(1\text{-}p)$, and $Var(X) = p/(1\text{-}p)^2$

Some Important Discrete Random Distributions

- Binomial

Out of *n* dice, exactly *k* dice have the same value: probability p ^{k} and *(n-k)* dice have different values: probability$(1\text{-}p)^{n\text{-}k}$.

For any *k* dice out of *n*:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k},$$

where,

$k=0,1,2,\ldots,n;\ n=0,1,2,\ldots;$ *p is the sucess probability, and*

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

Some Important Continuous Random Distributions

- Normal

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \text{ for } -\infty < x < \infty$$

and the cumulative distribution function can be obtained by

$$F_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{\frac{-(y-\mu)^2}{2\sigma^2}} dy$$

- $E[X] = \mu$, and $Var(X) = \sigma^2$

Some Important Continuous Random Distributions

- Uniform

$$f_X(x) = \begin{cases} \dfrac{d}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

and the cumulative distribution function is

$$F_X(x) = \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ 1, & \text{for x > b} \end{cases}$$

- $E[X] = (a+b)/2$, and $Var(X) = (b-a)^2/12$

Some Important Continuous Random Distributions

- Exponential

$$f_X(x) = \begin{cases} 0, & x < 0 \\ \lambda e^{-\lambda x}, & \text{for } 0 \le x < \infty \end{cases}$$

and the cumulative distribution function is

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & \text{for } 0 \le x < \infty \end{cases}$$

- $E[X] = 1/\lambda$, and $Var(X) = 1/\lambda^2$

Multiple Random Variables

- There are cases where the result of one experiment determines the values of several random variables
- The joint probabilities of these variables are:
  - Discrete variables:

    $$p(x_1, \ldots, x_n) = P(X_1 = x_1, \ldots, X_n = x_n)$$

  - Continuous variables:

    cdf: $$F_{x1x2\ldots xn}(x_1, \ldots, x_n) = P(X_1 \le x_1, \ldots, X_n \le x_n)$$

    pdf:

    $$f_{X_1, X_2, \ldots X_n}(x_1, x_2, \ldots x_n) = \frac{\partial^n F_{X_1, X_2, \ldots X_n}(x_1, x_2, \ldots x_n)}{\partial x_1 \partial x_2 \ldots \partial x_n}$$

Independence and Conditional Probability

- Independence: The random variables are said to be independent of each other when the occurrence of one does not affect the other. The pmf for discrete random variables in such a case is given by:

$$p(x_1,x_2,\ldots x_n)=P(X_1=x_1)P(X_2=x_2)\ldots P(X_3=x_3)$$

and for continuous random variables as:

$$F_{X1,X2,\ldots Xn} = F_{X1}(x_1)F_{X2}(x_2)\ldots F_{Xn}(x_n)$$

Important Properties of Random Variables

- Sum property of the expected value
  - Expected value of the sum of random variables:

$$E\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i E[X_i]$$

- Product property of the expected value
  - Expected value of product of stochastically independent random variables

$$E\left[\prod_{i=1}^{n} X_i\right] = \prod_{i=1}^{n} E[X_i]$$

Important Properties of Random Variables

- Sum property of the variance
  - Variance of the sum of random variables is

$$Var\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 Var(X_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} a_i a_j \operatorname{cov}[X_i, X_j]$$

where $cov[X_i,X_j]$ is the covariance of random variables X_i and X_j

and

$$\begin{aligned}\operatorname{cov}[X_i, X_j] &= E[(X_i - E[X_i])(X_j - E[X_j])] \\ &= E[X_i X_j] - E[X_i]E[X_j]\end{aligned}$$

If random variables are independent of each other, i.e., $cov[X_i,X_j]=0$, then

$$Var\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 Var(X_i)$$

Important Properties of Random Variables

- For a special case $Z = X+Y$; If both X and Y are non negative random variables, then pdf is the convolution of the individual pdfs, $f_X(x)$ and $f_Y(y)$.

$$f_Z(z) = \int_0^z f_X(x) f_Y(z-x)\,dx, \quad \text{for } -\infty \le z < \infty$$

Central Limit Theorem

The *Central Limit Theorem* states that whenever a random sample (X_1, X_2,.. X_n) of size n is taken from any distribution with expected value $E[X_i] = \mu$ and variance $Var(X_i) = \sigma^2$, where i =1,2,..,n, then their arithmetic mean is defined by

$$S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Central Limit Theorem

- The sample mean is approximated to a normal distribution with
  - $E[S_n] = \mu$, and
  - $Var(S_n) = \sigma^2 / n$.
- The larger the value of the sample size n, the better the approximation to the normal.
- This is very useful when interference between signals needs to be considered.

Poisson Process and its Properties

Random Process

- A Random process is a sequence of events "randomly spaced in time".
- For example, customers arriving at a bank are similar to packets arriving at a buffer.

Poisson Process

- A stochastic process *A(t)* (t > 0, *A(t)* >=0) is said to be a Poisson process with rate λ if

1. *A(t)* is a counting process that represents the total number of arrivals in [0, *t*]
2. The numbers of arrivals that occur in disjoint intervals are independent
3. The number of arrivals in any [t, $t + \tau$] is Poisson distributed with parameter $\lambda\tau$

$$P\{A(t+\tau) - A(t) = n\} = e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!}, \quad n = 0,1,\ldots$$

Properties of Poisson Process (1)

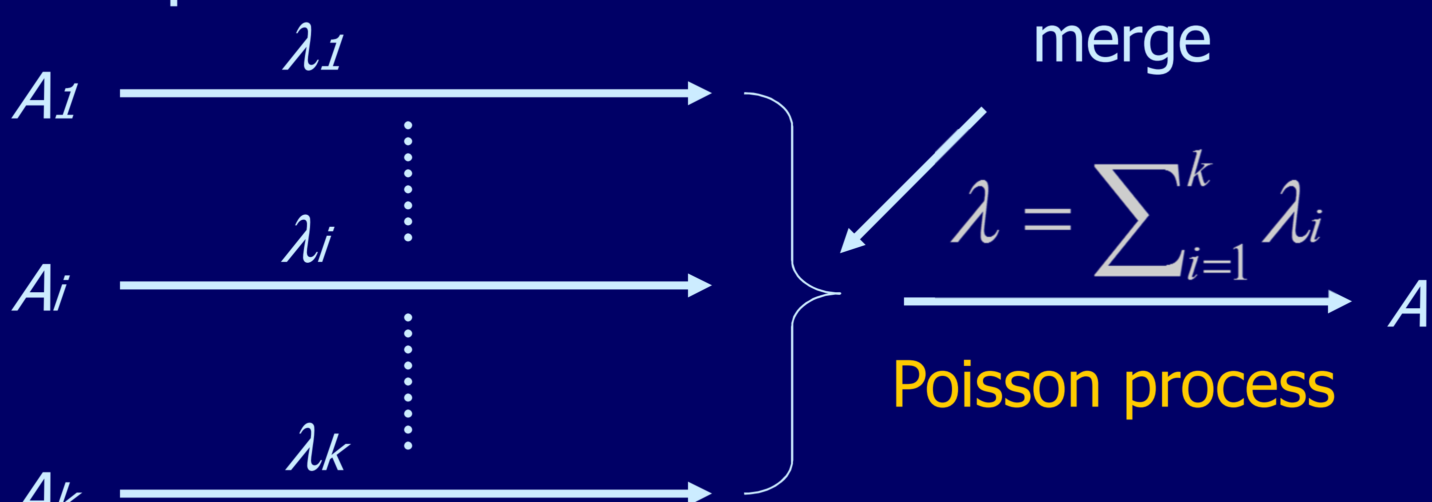
- Interarrival times τ_n are independent and exponentially distributed with parameter λ

$$P\{\tau_n \le s\} = 1 - e^{-\lambda s}, \quad s \ge 0$$

- The mean and variance of interarrival times τ_n are $1/\lambda$ and $1/\lambda^2$, respectively

Properties of Poisson Process (2)

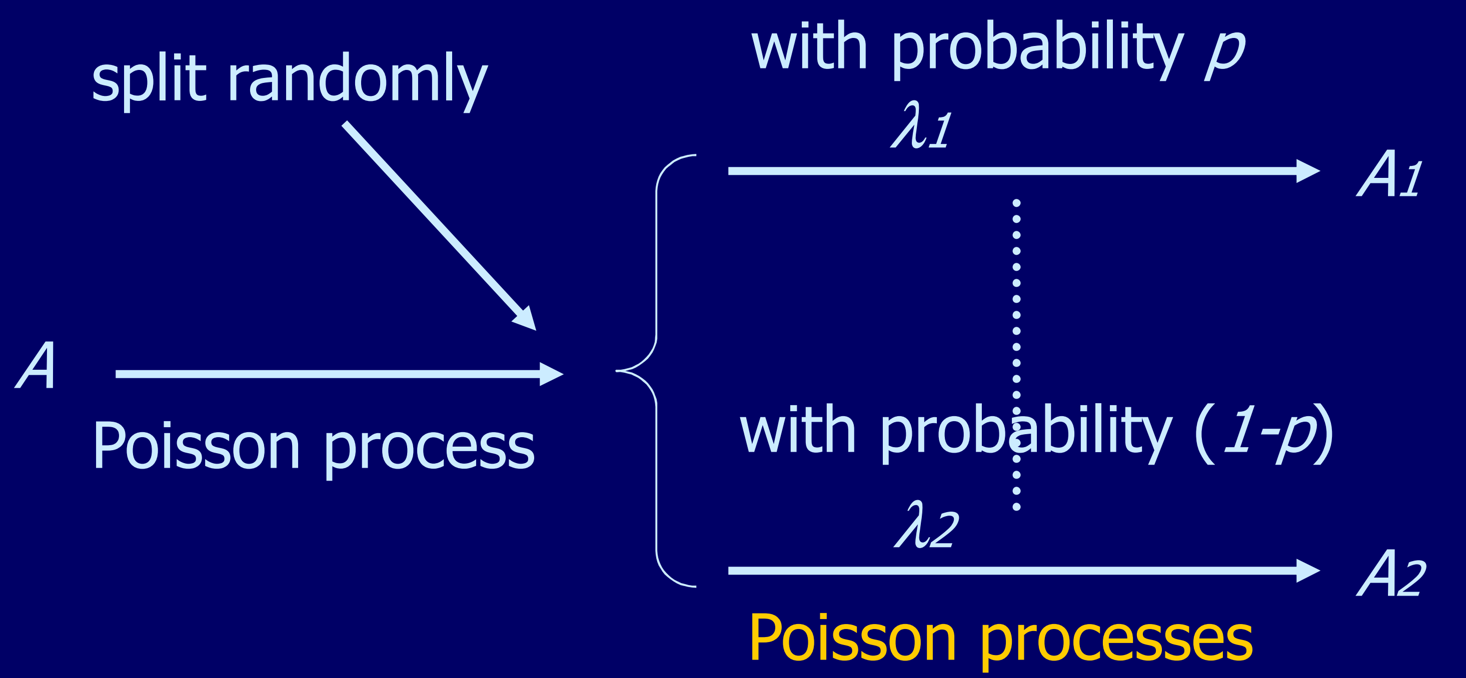
- If two or more independent Poisson process $A_1, \ldots, A_k$ are merged into a single process $A = A_1 + A_2 + \ldots + A_k$, the process A is Poisson with a rate equal to the sum of the rates of its components



independent Poisson processes

Properties of Poisson Process (3)

- If a Poisson process A is split into two other processes A_1 and A_2 by randomly assigning each arrival to A_1 or A_2, processes A_1 and A_2 are Poisson



Traffic Generation -- Poisson Process

- Generate Random Inter-arrival times that are exponentially distributed. Note that Exponentially Distributed Inter-arrival times can be generated from a Uniform distribution U(0,1) as follows:

Y = -(1/lambda) * ln(u(0,1))

Y is an exponentially distributed random number With parameter lambda.

Gauss-Markov Mobility Model

$$s_n = \alpha s_{n-1} + (1-\alpha)\bar{s} + \sqrt{(1-\alpha^2)} s_{x_{n-1}}$$

$$d_n = \alpha d_{n-1} + (1-\alpha)\bar{d} + \sqrt{(1-\alpha^2)} d_{x_{n-1}}$$

where s_n and d_n are the new speed and direction of the mobile node at time interval n;
α, $0 \le \alpha \le 1$, is the tuning parameter used to vary the randomness;
S_bar, d_bar are constants representing the mean value of speed and direction
And $s_{x_{n-1}}, d_{x_{n-1}}$ are random variables from a Gaussian distribution.

$$x_n = x_{n-1} + s_{n-1} \cos d_{n-1}$$

$$y_n = y_{n-1} + s_{n-1} \sin d_{n-1}$$

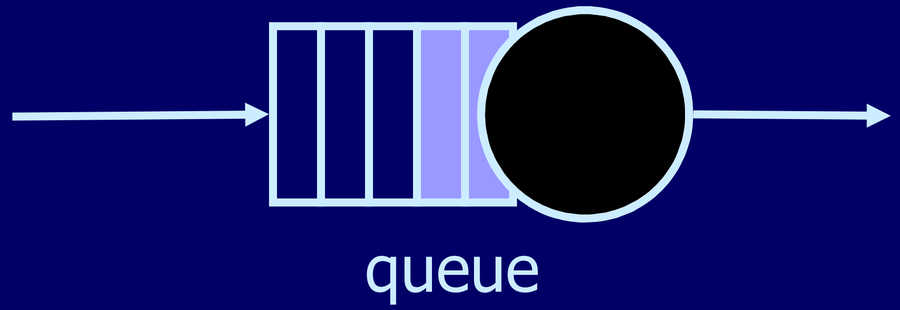
where (x_n, y_n) and (x_{n-1}, y_{n-1}) are the x and y coordinates of the mobile node's position at the n^{th} and $(n-1)^{st}$ time intervals, respectively.
s_{n-1} and d_{n-1} are the speed and direction of the mobile node, respectively, at the $(n-1)^{st}$ time interval

Introduction to Queueing Theory

Based on the slides of Prof. Hiroyuki Ohsaki
Graduate School of Information Science & Technology, Osaka University, Japan

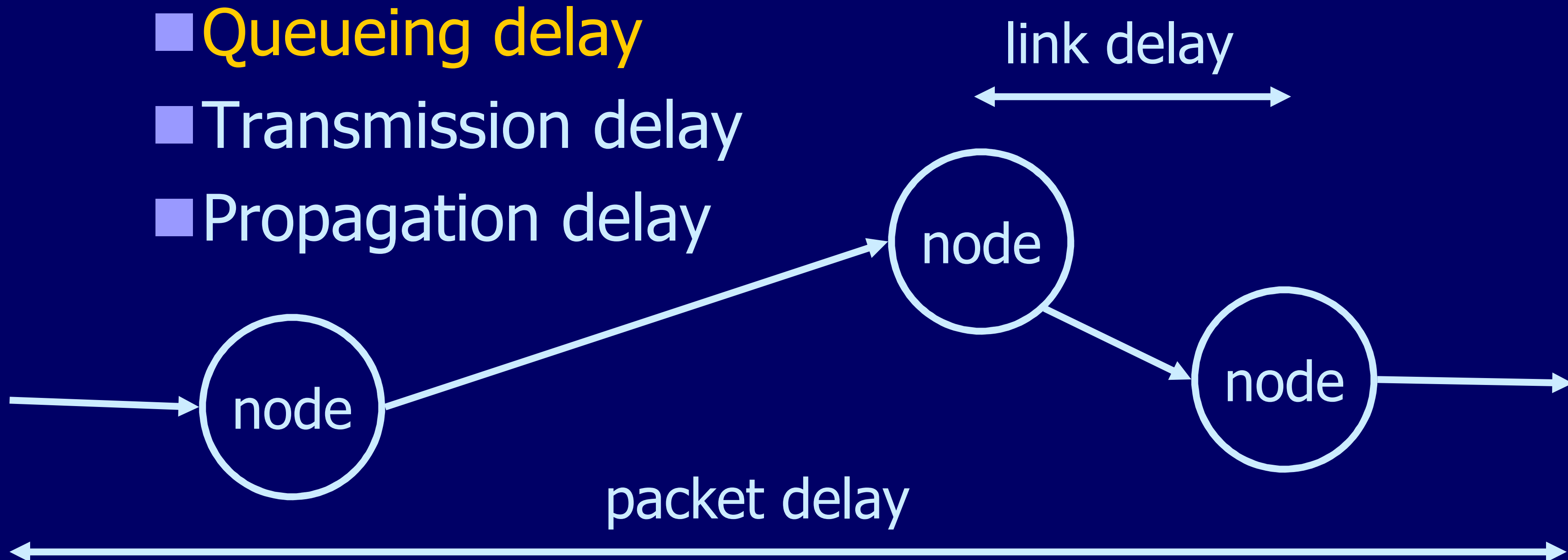
What is Queueing Theory?

- Primary methodological framework for analyzing network delay
- Often requires simplifying assumptions since realistic assumptions make meaningful analysis extremely difficult
- Provide a basis for adequate delay approximation



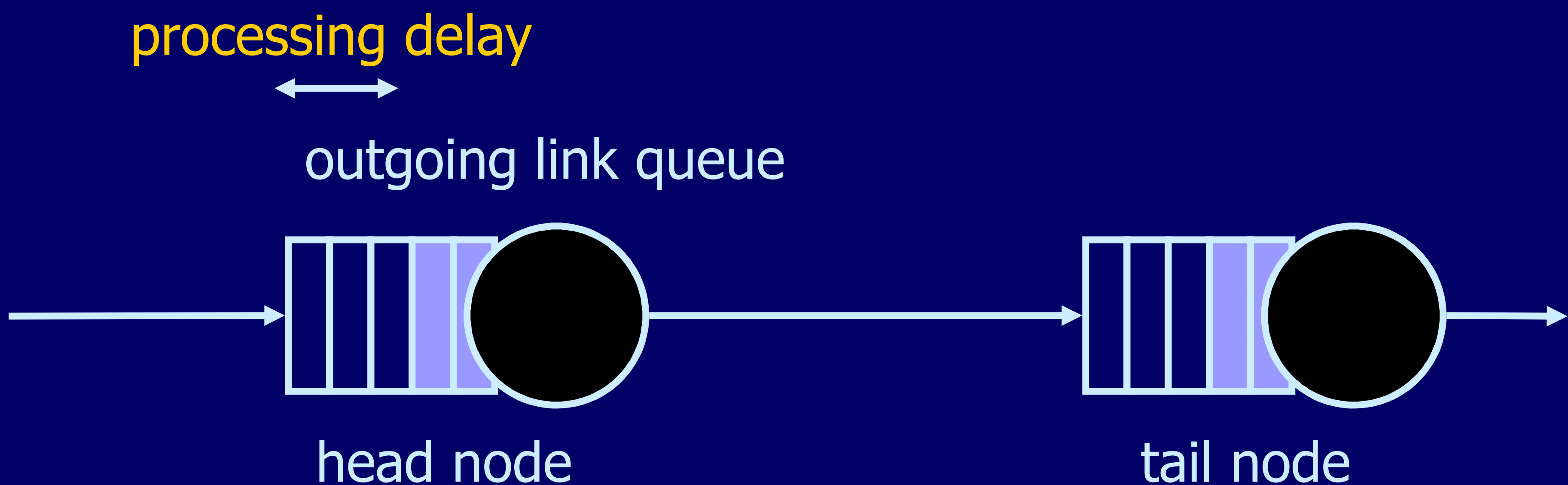
Packet Delay

- Packet delay is the sum of delays on each subnet link traversed by the packet
- Link delay consists of:
  - Processing delay
  - Queueing delay
  - Transmission delay
  - Propagation delay



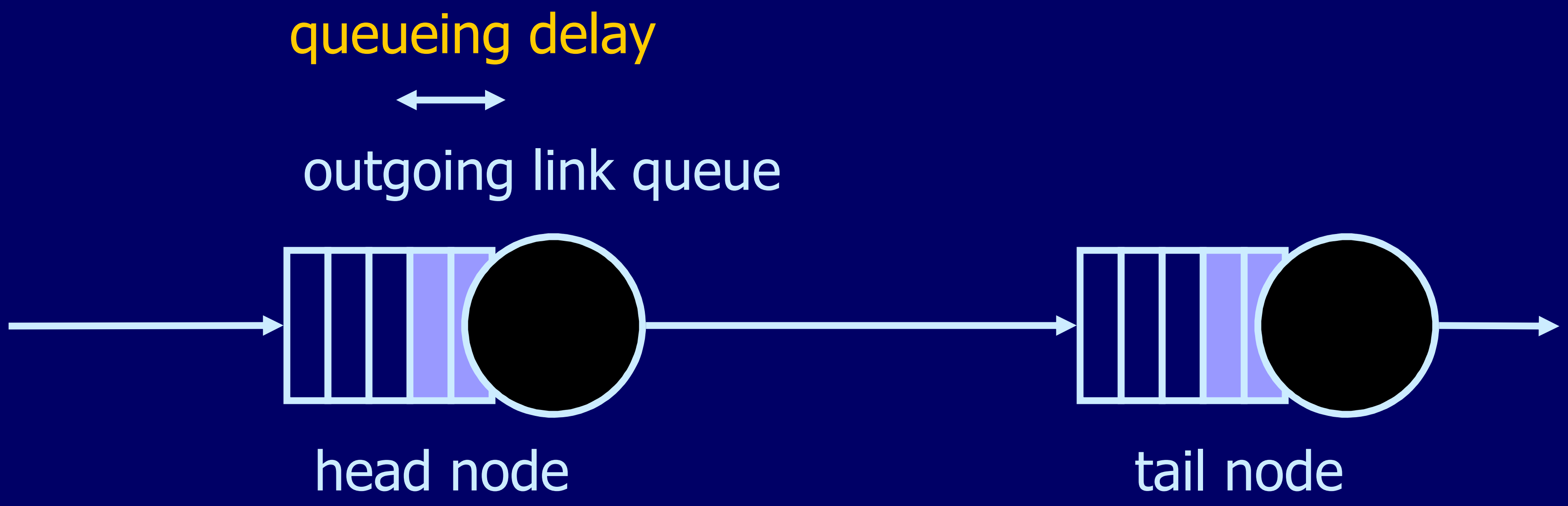
Link Delay Components (1)

- Processing delay
  - Delay between the time the packet is correctly received at the head node of the link and the time the packet is assigned to an outgoing link queue for transmission



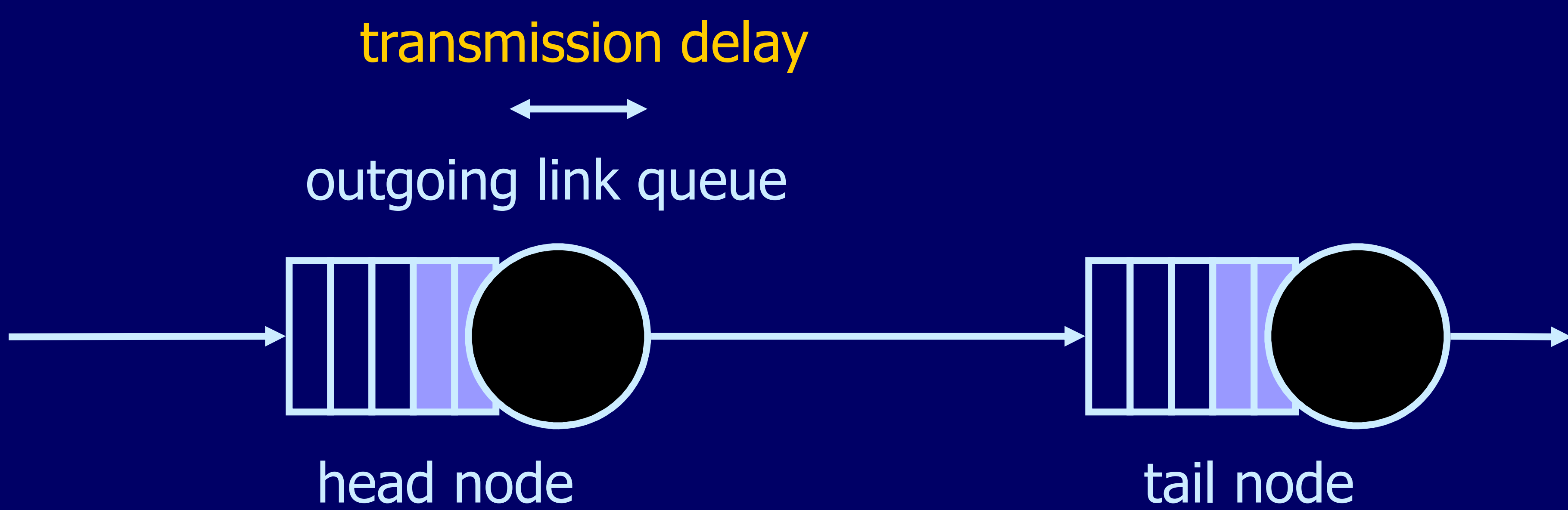
Link Delay Components (2)

- Queueing delay
  - Delay between the time the packet is assigned to a queue for transmission and the time it starts being transmitted



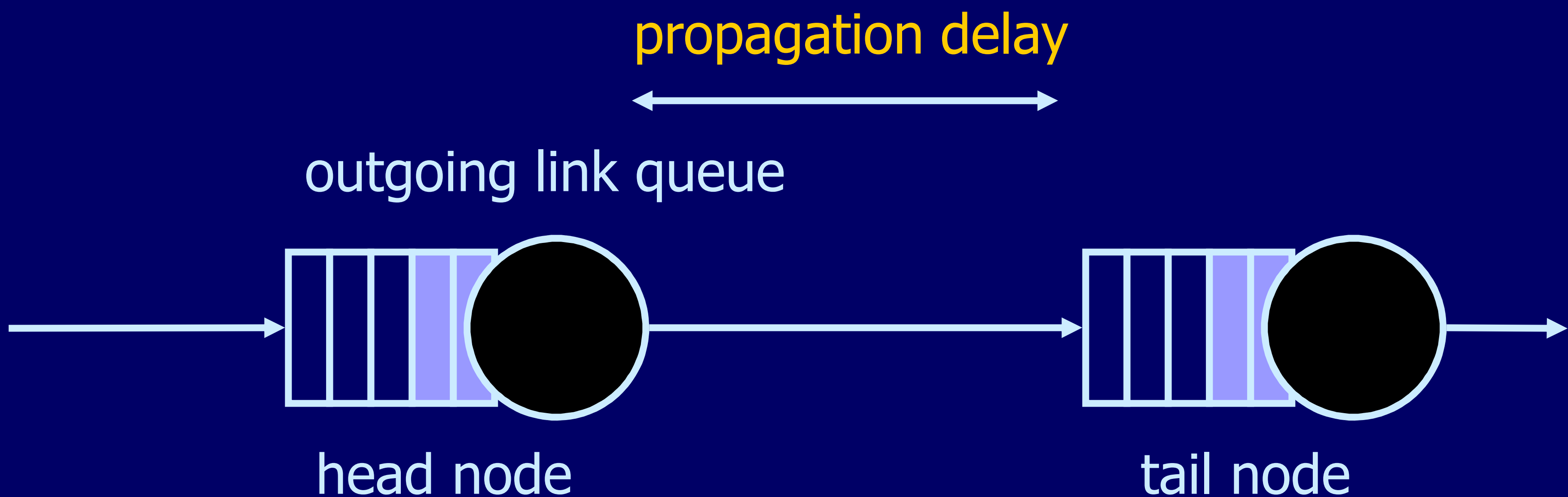
Link Delay Components (3)

- Transmission delay
  - Delay between the times that the first and last bits of the packet are transmitted



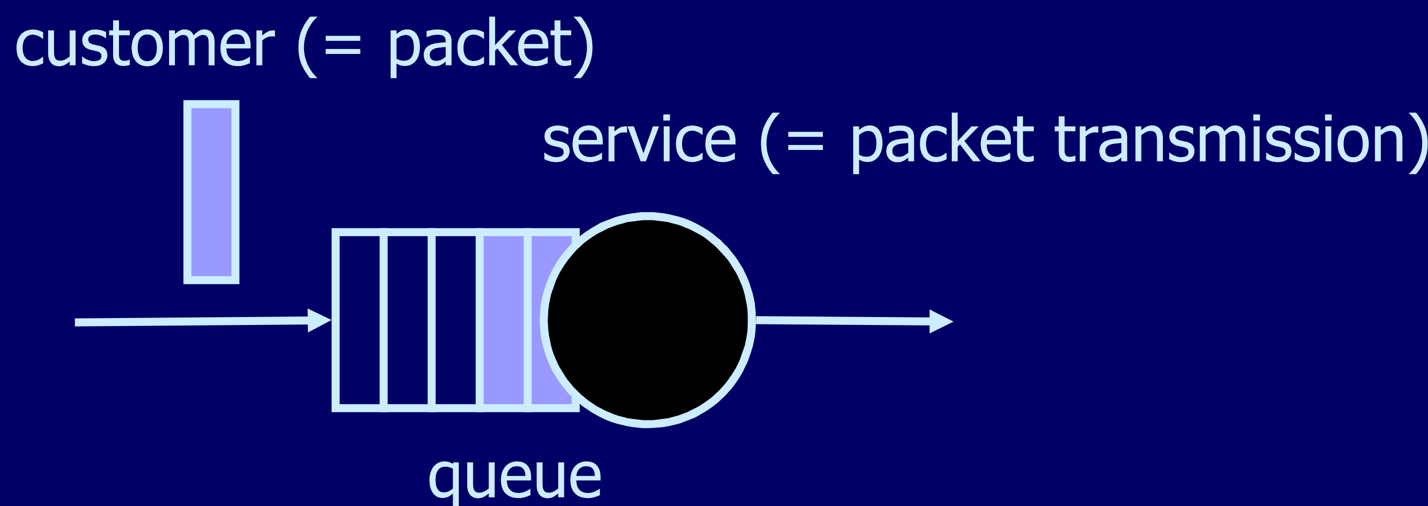
Link Delay Components (4)

- Propagation delay
  - Delay between the time the last bit is transmitted at the head node of the link and the time the last bit is received at the tail node



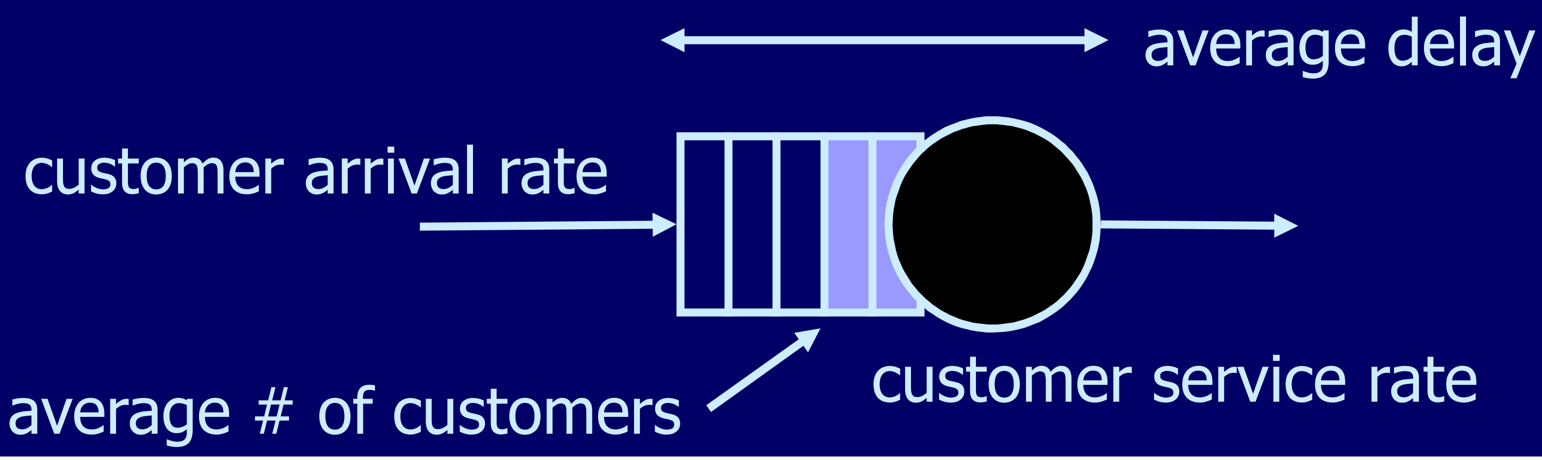
Queueing System (1)

- Customers (= packets) arrive at random times to obtain service
- Service time (= transmission delay) is L/C
  - L: Packet length in bits
  - C: Link transmission capacity in bits/sec



Queueing System (2)

- Assume that we already know:
  - Customer arrival rate
  - Customer service rate
- We want to know:
  - Average number of customers in the system
  - Average delay per customer



Queueing Networks

- Complex systems can be modeled as a queueing networks.
- Examples:
  - Analysis of the delay performance of REST web services installed on web farms.
  - Analysis of the delay performance of the Message Queuing Telemetry Transport (MQTT) in the context of IoT.
  - Publish/Subscribe Pattern (push) vs. pull models.

Little's Theorem

Definition of Symbols (1)

- p_n = Steady-state probability of having n customers in the system
- λ = Arrival rate (inverse of average interarrival time)
- μ = Service rate (inverse of average service time)
- N = Average number of customers in the system

Definition of Symbols (2)

- N_Q = Average number of customers waiting in queue
- T = Average customer time in the system
- W = Average customer waiting time in queue (does not include service time)
- S = Average service time

Little's Theorem

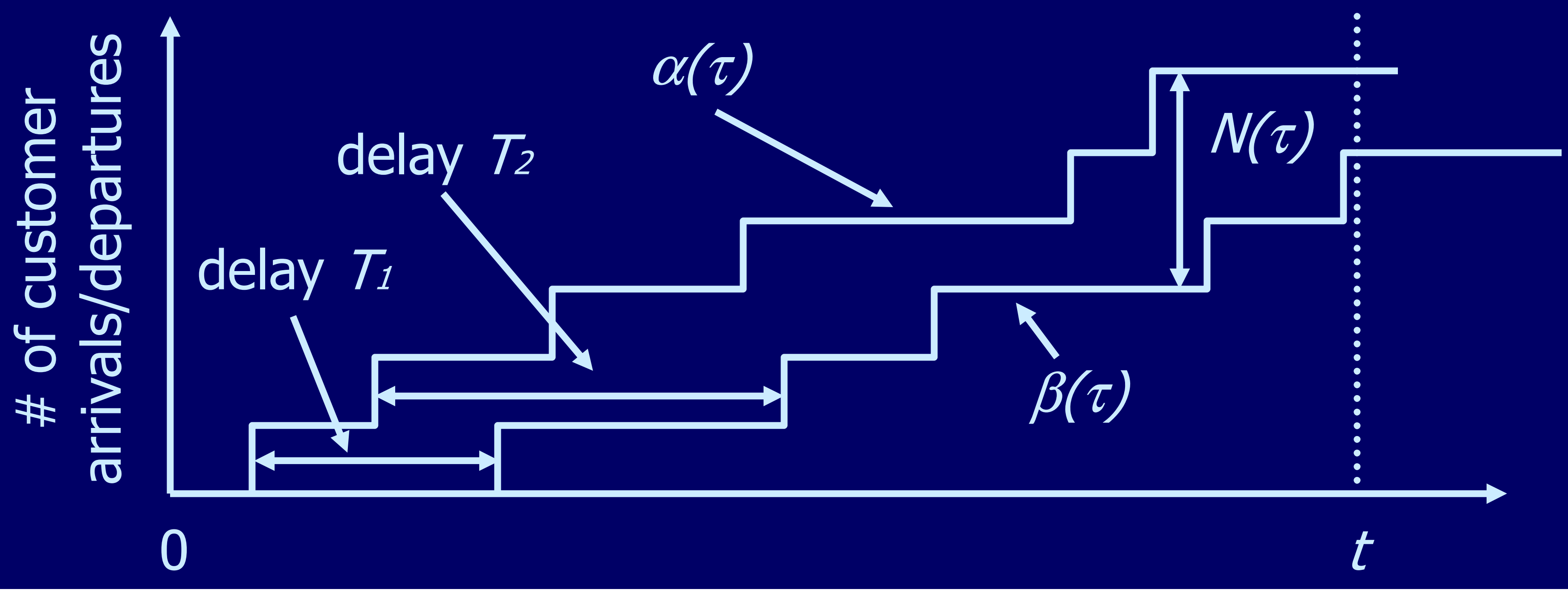
- N = Average number of customers
- λ = Arrival rate
- T = Average customer time

$$N = \lambda T$$

- Hold for almost every queueing system that reaches a steady-state
- Express the natural idea that crowded systems (large N) are associated with long customer delays (large T) and reversely

Illustration of Little's Theorem

- Assumption:
  - The system is initially empty
  - Customers depart from the system in the order they arrive



Application of Little's Theorem (1)

- N_Q = Average # of customers waiting in queue
- W = Average customer waiting time in queue

$$N_Q = \lambda W$$

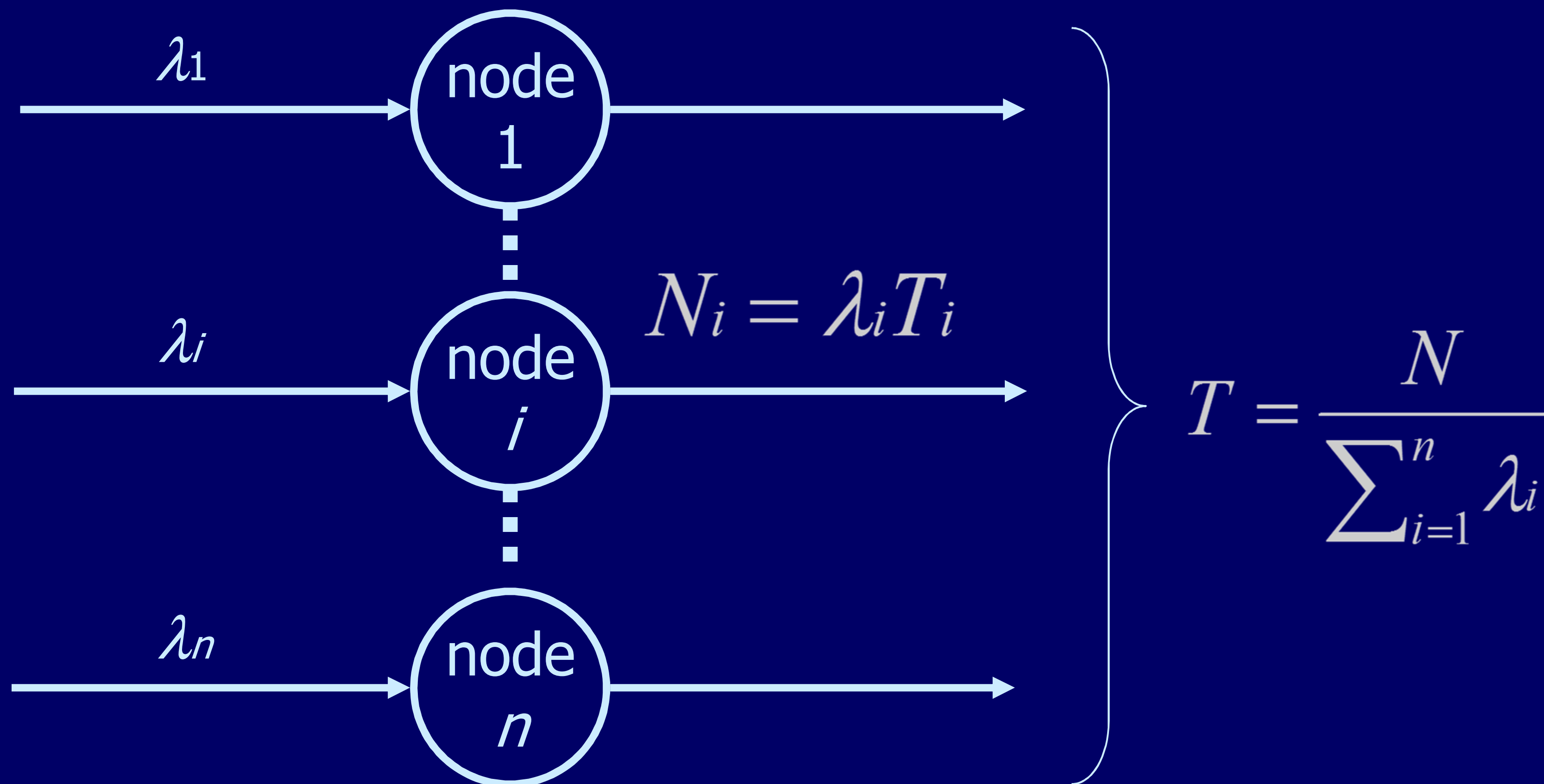
- X = Average service time
- ρ = Average # of packets under transmission

$$\rho = \lambda X$$

- ρ is called the utilization factor (= the proportion of time that the line is busy transmitting a packet)

Application of Little's Theorem (2)

- λ_i = Packet arrival rate at node *i*
- *N* = Average total # of packets in the network



Little’s Theorem: Problem

- Customers arrive at a fast-food restaurant as a Poisson process with an arrival rate of 5 per min
- Customers wait at a cash register to receive their order for an average of 5 min
- Customers eat in the restaurant with probability 0.5 and carry out their order without eating with probability 0.5
- A meal requires an average of 20 min
- What is the average number of customers in the restaurant? (Answer: 75)

Standard Notation of Queueing Systems

Standard Notation of Queueing Systems (1)

X/Y/Z/K

- *X* indicates the nature of the arrival process
  - *M*: Memoryless (= Poisson process, exponentially distributed interarrival times)
  - *G*: General distribution of interarrival times
  - *D*: Deterministic interarrival times

Standard Notation of Queueing Systems (2)

X/Y/Z/K

- *Y* indicates the probability distribution of the service times
  - *M*: Exponential distribution of service times
  - *G*: General distribution of service times
  - *D*: Deterministic distribution of service times

Standard Notation of Queueing Systems (3)

X/Y/Z/K

- *Z* indicates the number of servers
- *K* (optional) indicates the limit on the number of customers in the system
- Examples:
  - M/M/1, M/M/m, M/M/∞, M/M/m/m
  - M/G/1, G/G/1
  - M/D/1, M/D/1/m

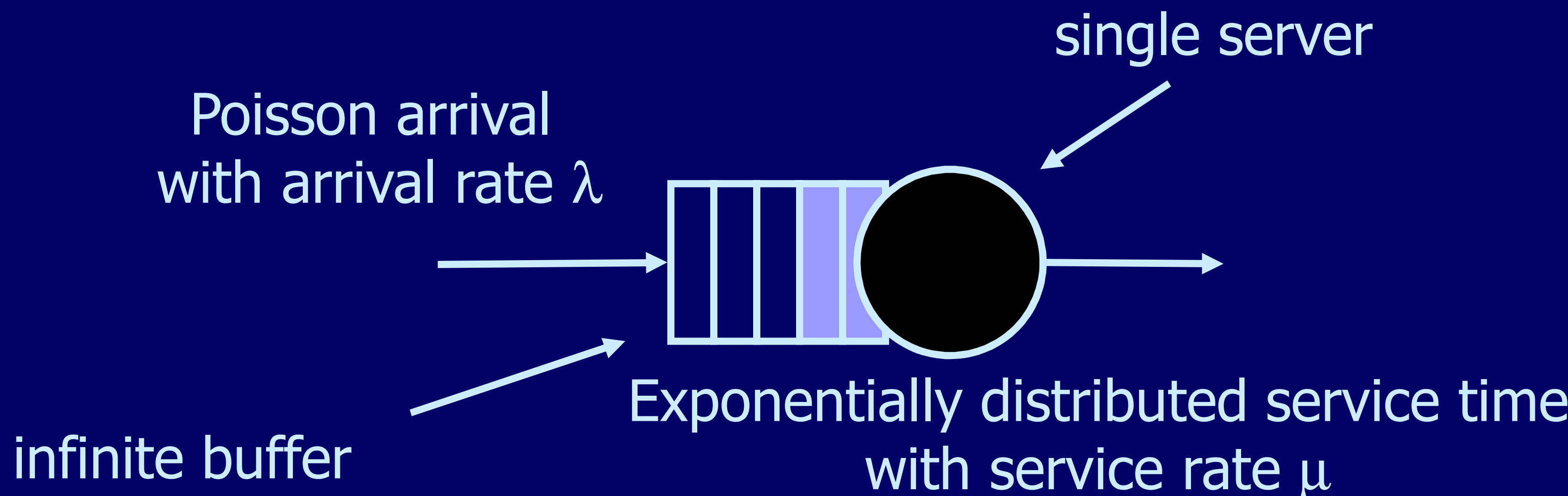
Scheduling Disciplines

- First Come First Serve (FCFS or FIFO)
- Round Robin
  - Work Conserving vs. non-work conserving
- Processor Sharing: clients or jobs are all served simultaneously, each receiving an equal fraction of the service capacity available
  - Generalized Processor Sharing: weighted processor sharing
- Fair Queueing: allow multiple packet flows to fairly share the link capacity
  - Weighted Fair Queueing: Each data flow has a separate FIFO queue
- Preemptive Priority Scheduling: A job in service is stopped if a higher priority job arrives. The preempted job may resume later (work conserving).
  - Non-Preemptive Priority Scheduling: A job in service is always completed

M/M/1 Queueing System

M/M/1 Queueing System

- A single queue with a single server
- Customers arrive according to a Poisson process with rate λ
- The probability distribution of the service time is exponential with mean $1/\mu$



M/M/1 Queueing System: Results (1)

- Utilization factor (proportion of time the server is busy)

$$\rho = \frac{\lambda}{\mu}$$

- Probability of *n* customers in the system

$$p_n = \rho^n (1 - \rho)$$

- Average number of customers in the system

$$N = \frac{\rho}{1 - \rho}$$

M/M/1 Queueing System: Results (2)

- Average customer time in the system

$$T = \frac{N}{\lambda} = \frac{1}{\mu - \lambda}$$

- Average number of customers in queue

$$N_Q = \lambda W = \frac{\rho^2}{1 - \rho}$$

- Average waiting time in queue

$$W = T - \frac{1}{\mu} = \frac{\rho}{\mu - \lambda}$$

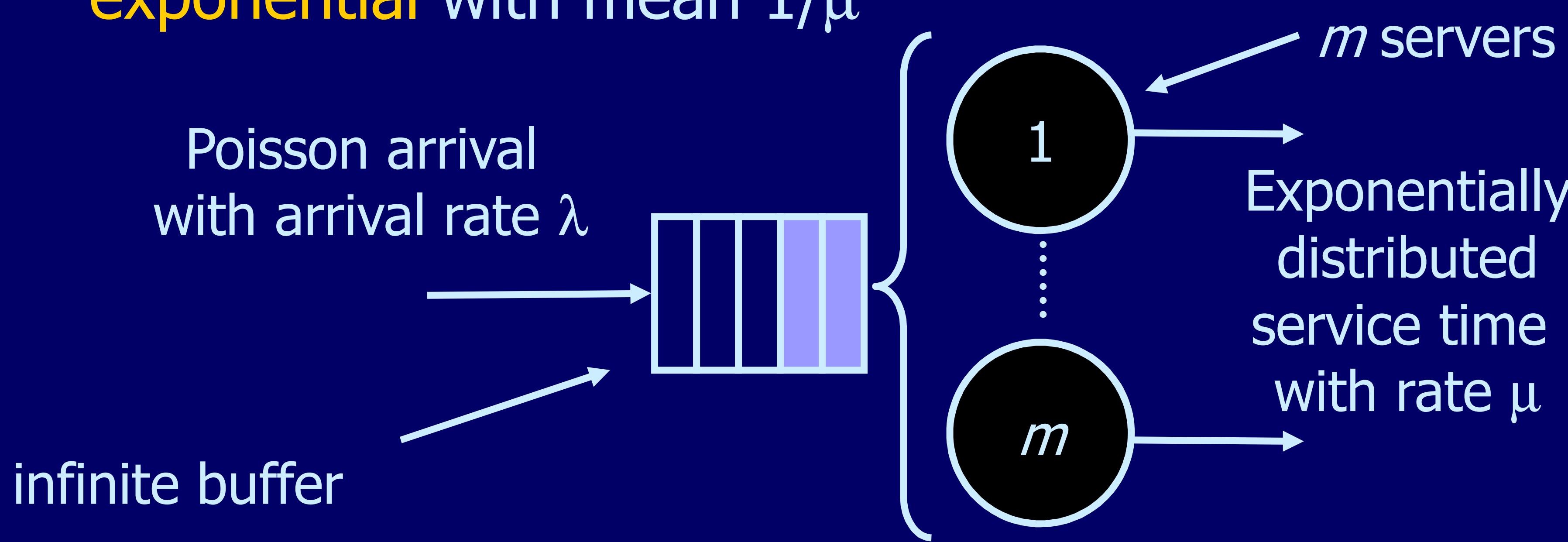
M/M/1 Queueing System: Problem

- Customers arrive at a fast-food restaurant as a Poisson process with an arrival rate of 5 per min
- Customers wait at a cash register to receive their order for an average of 5 minutes
- Service times to customers are independent and exponentially distributed
- What is the average service rate at the cash register? (Answer: 5.2)
- If the cash register serves 10% faster, what is the average waiting time of customers? (Answer: 1.39min)

M/M/m Queueing System

M/M/m Queueing System

- A single queue with *m* servers
- Customers arrive according to a Poisson process with rate λ
- The probability distribution of the service time is exponential with mean $1/\mu$



M/M/m Queueing System: Results (1)

- Ratio of arrival rate to maximal system service rate

$$\rho = \frac{\lambda}{m\mu}$$

- Probability of *n* customers in the system

$$p_0 = \left[\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!(1-\rho)} \right]^{-1}$$

$$p_n = \begin{cases} p_0 \dfrac{(m\rho)^n}{n!} & n \le m \\ p_0 \dfrac{m^m \rho^m}{m!} & n > m \end{cases}$$

M/M/m Queueing System: Results (2)

- Probability that an arriving customer has to wait in queue (*m* customers or more in the system)

$$P_Q = \frac{p_0 (m\rho)^m}{m!(1-\rho)}$$

- Average waiting time in queue of a customer

$$W = \frac{N_Q}{\lambda} = \frac{\rho P_Q}{\lambda(1-\rho)}$$

- Average number of customers in queue

$$N_Q = \sum_{n=0}^{\infty} n p_{m+n} = \frac{\rho P_Q}{1-\rho}$$

M/M/m Queueing System: Results (3)

- Average customer time in the system

$$T = \frac{1}{\mu} + W = \frac{1}{\mu} + \frac{P_Q}{m\mu - \lambda}$$

- Average number of customers in the system

$$N = \lambda T = m\rho + \frac{\rho P_Q}{1 - \rho}$$

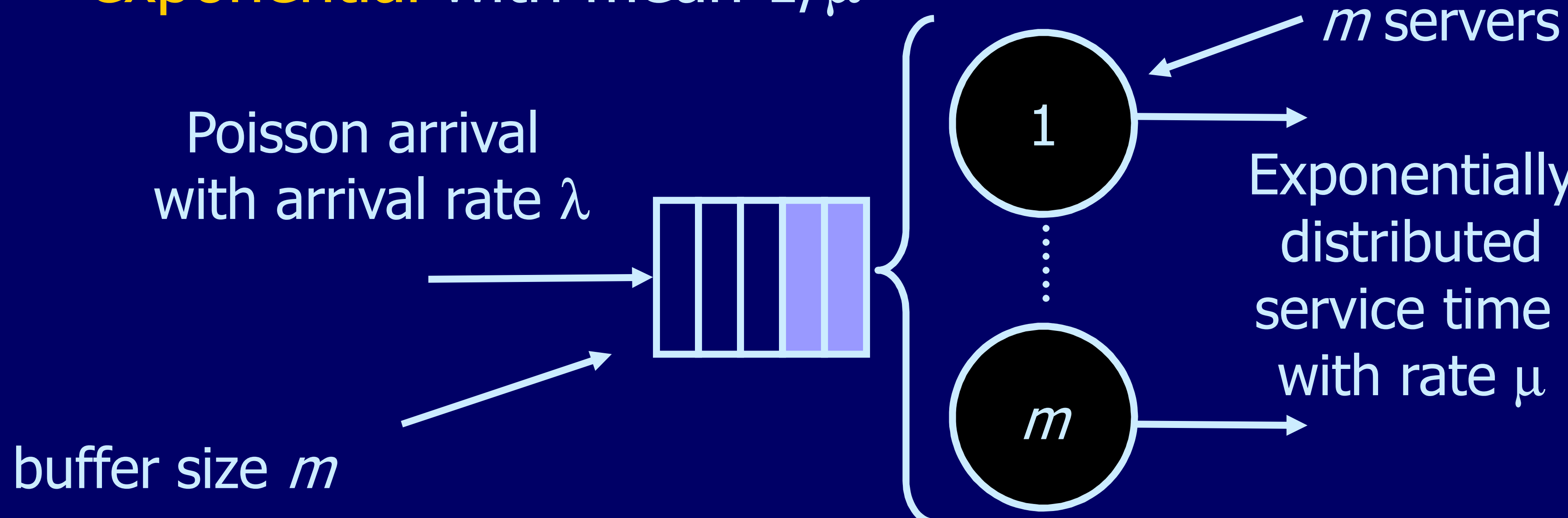
M/M/m Queueing System: Problem

- A mail-order company receives calls at a Poisson rate of 1 per 2 min
- The duration of the calls is exponentially distributed with mean 2 min
- A caller who finds all telephone operators busy patiently waits until one becomes available
- The number of operators is 2 on weekdays or 3 on weekend
- What is the average waiting time of customers in queue? (Answer: 0.67min and 0.09min)

M/M/m/m Queueing System

M/M/m/m Queueing System

- A single queue with *m* servers (buffer size *m*)
- Customers arrive according to a Poisson process with rate λ
- The probability distribution of the service time is exponential with mean $1/\mu$



M/M/m/m Queueing System: Results

- Probability of *m* customers in the system

$$p_0 = \left[\sum_{n=0}^{m} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}\right]^{-1}$$

$$p_n = p_0 \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}, \quad n = 1, 2, \ldots, m$$

- Probability that an arriving customer is lost (Erlang B Formula)

$$p_m = \frac{(\lambda / \mu)^m / m!}{\sum_{n=0}^{m} (\lambda / \mu)^n / n!}$$

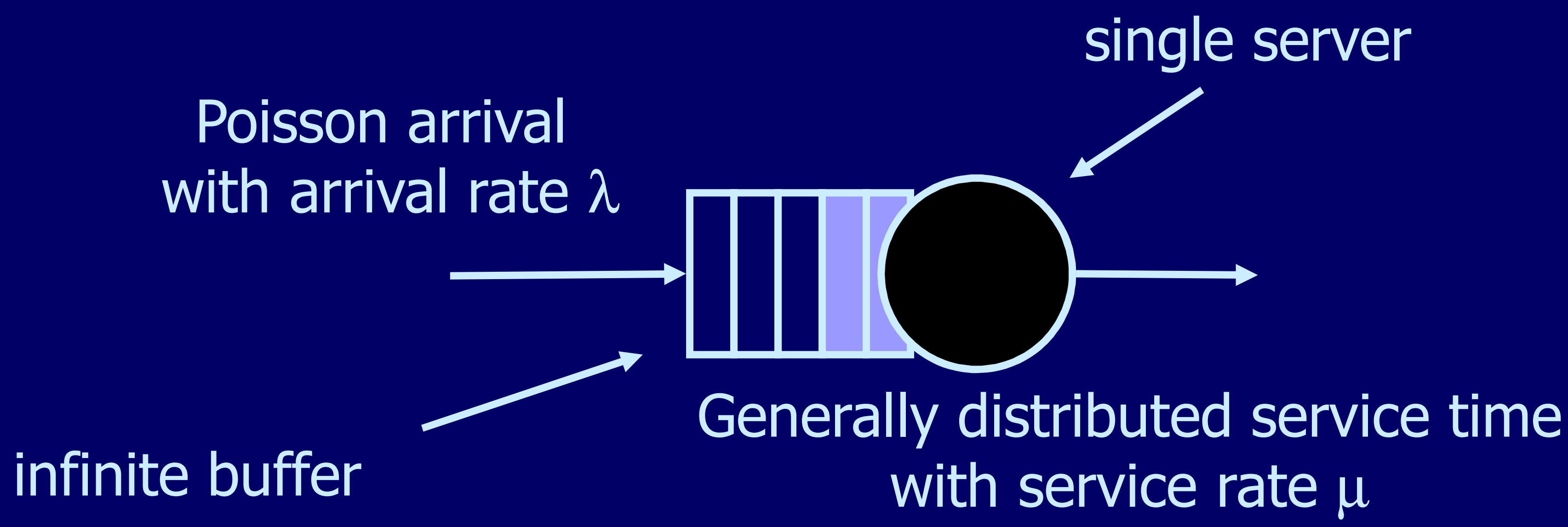
M/M/m/m Queueing System: Problem

- A telephone company establishes a direct connection between two cities expecting Poisson traffic with rate 0.5 calls/min
- The durations of calls are independent and exponentially distributed with mean 2 min
- Interarrival times are independent of call durations
- How many circuits should the company provide to ensure that an attempted call is blocked with probability less than 0.1? (Answer: 3)

M/G/1 Queueing System

M/G/1 Queueing System

- A single queue with a single server
- Customers arrive according to a Poisson process with rate λ
- The mean and second moment of the service time are $1/\mu$ and X_2



M/G/1 Queueing System: Results (1)

- Utilization factor

$$\rho = \frac{\lambda}{\mu}$$

- Mean residual service time

$$R = \frac{\lambda X_2}{2}$$

M/G/1 Queueing System: Results

- Pollaczek-Khinchin formula

$$W = \frac{R}{1-\rho} = \frac{\lambda X_2}{2(1-\rho)}$$

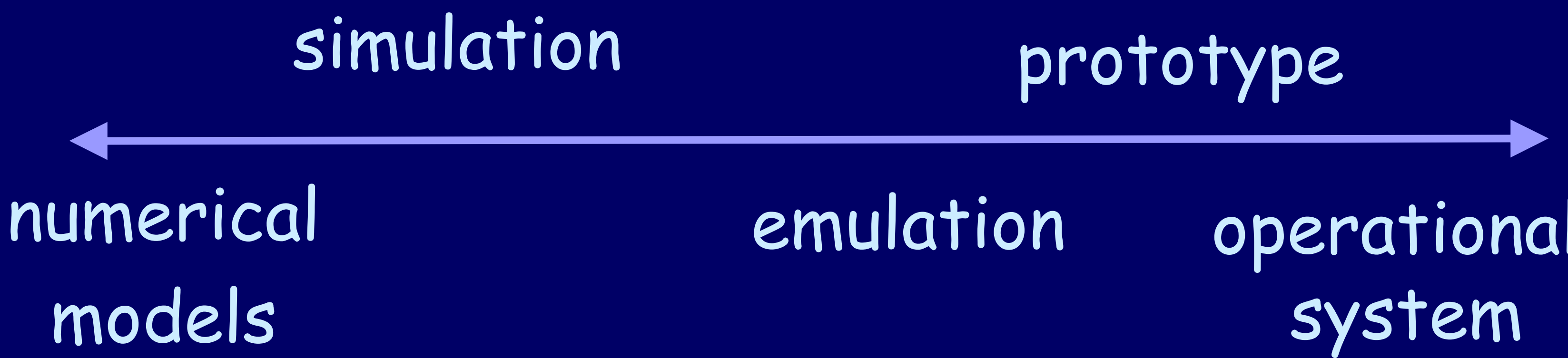
$$T = \frac{1}{\mu} + W$$

$$N_Q = \lambda W = \frac{\lambda^2 X_2}{2(1-\rho)}$$

$$N = \lambda T = \rho + \frac{\lambda^2 X_2}{2(1-\rho)}$$

Simulation

The System evaluation spectrum



What is simulation?

External inputs
to system
(the environment)

system boundary

system under study
(has deterministic rules
governing its behavior)

observer

"real" life

psuedo random inputs
to system
(models environment)

program boundary

computer program
simulates deterministic
rules governing behavior

observer

"simulated" life

Why Simulation?

- *goal:* study system *performance, operation*
- real-system not *available, is complex/costly or dangerous* (eg: space simulations, flight simulations)
- quickly evaluate design *alternatives* (eg: different system configurations)
- evaluate *complex functions* for which closed form formulas or numerical techniques not available

Programming a simulation

What 's in a simulation program?

- *simulated time:* internal (to simulation program) variable that keeps track of simulated time
- *system "state":* variables maintained by simulation program define system "state"
  - e.g., may track number (possibly order) of packets in queue, current value of retransmission timer
- *events:* points in time when system changes state
  - each event has associated *event time*
    - e.g., arrival of packet to queue, departure from queue
    - precisely at these points in time that simulation must take action (change state and may cause new future events)
  - model for time between events (probabilistic) caused by external environment

Discrete Event Simulation

- simulation program maintains and updates list of future events: event list
- simulator structure:

Need:

- well defined set of events
- for each event: simulated system action, updating of event list

Conclusion

- Queueing models provide qualitative insights on the performance of computer networks, and quantitative predictions of average packet delay
- To obtain tractable queueing models for computer networks, it is frequently necessary to make simplifying assumptions
- A more accurate alternative is simulation, which, however, can be slow, expensive, and lacking in insight